Strategies to improve problem-solving skills

Irene Stone
Mathematics Teacher
St. Mark’s Community School
Two strategies:

• How working collaboratively through an online environment can support problem-solving skills of students

• Weekly puzzle competition – whole school
What is problem solving?

• “Problems are situations with no obvious solution, and solving problems requires thinking and learning in action” (OECD, 2014, p. 26).

• Problem solving is “concerned with non-routine tasks” where one “can’t apply previously learned procedures” (OECD, 2014, p. 30). The chief examiner reports recommend that teachers engage students in non-routine tasks (SEC, 2015a, 2015b).
Polya’s 4 step Problem Solving process

1. Understand the Problem
   - Identifiable
   - Non Routine
   - Knowledge Base
   - Describe using words

2. Devising a Plan
   - Attempting the problem
   - Using words to explain what it means

3. Carrying out the Plan
   - Multi-representational
   - Explaining
   - Collaborative
   - Scaffolding

4. Looking Back
   - Justifying using words
   - Checking answers
   - Perseverance
   - Try again

(Polya, 1957, p.xvii)
Theory – themes that emerged from literature

• **Reflecting and trying again** are key elements of problem solving (DES, 2013; Hurme & Järvelä, 2005; Kim & Hannafin, 2011)

• When problem solving, one should **use words** to describe, explain, or justify what they’re doing (Hurme & Jarvela, 2005; Muir et al., 2008; SEC, 2015a).

• Students should aim for a **multi-representational** approach when problem solving (Boaler, 2010; Schoenfeld, 1992).

• Problem solving should be **collaborative** (Kim & Hannafin, 2011; Tanner & Jones, 2000).

• **Scaffolding** is where teachers do not tell students too much but instead guide them by asking stimulating questions; scaffolding can enhance problem-solving skills (Polya, 1957, p. v; Tan Yeen-Ju, Mai, & Selvaretnam, 2015, p. 845; Tanner & Jones, 2000, p. 29).
Background

- Problem Solving (PS) - an integral part of mathematical learning (Boaler, 2010; Polya, 1957).

- Problem-solving skills in demand in today’s workplace and in education (OECD, 2014, p. 26).

- Project Maths course was introduced in 2008. One of the aims of the new syllabus was to embed problem-solving skills amongst students; to steer them away from the procedural approach which dominated the “traditional” mathematics classroom (NCCA, 2005, p. 18).

- A need for collaboration and digital skills, amongst others, to be embedded in the new Junior Cycle (DES, 2015a, p. 7).

- One of the key points of the Digital Strategy for Schools is to embed digital skills within all subjects of the curriculum (DES, 2015b).

- The Minister of Education in 2015, Jan O’ Sullivan, encouraged all teachers to use technology “to give learners the tools to collaborate and to examine engaging problems”. 

The problem

• Since the introduction of Project Maths, reports suggest that students still lacking in PS skills, in particular in problems involving algebra – Chief Examiner Reports, PISA 2012 and PISA 2015 results (Perkins & Shiel, 2014, p. 7; SEC, 2015a, 2015b).

• Latest, PISA 2015 - students are still struggling with “higher-order” skills such as problem solving (DES, 2016, p.11).

• Research suggests that students’ exposure to problem-solving skills in a classroom environment is limited (Bray & Tangney, 2016; Kim & Hannafin, 2011; Schoenfeld, 1992).

• Teachers feel pressure of terminal exam and are therefore reluctant to facilitate methodologies such as group-work and setting open-ended tasks (NCCA, 2012, p. 11)
The research

• This research investigated if working collaboratively through an online environment can support problem-solving skills.

• Google Docs, the online environment used for this research, allows for collaborative editing of a document.

• An exploratory case study was carried out which involved 30 post-primary students from a 2\textsuperscript{nd} year Mathematics class, ages 13-14 years. Working in groups of 4 or 5 they were given problems to solve on the Google Doc environment over a two week period.

• The research employed mixed methodologies.

• Findings of the research concluded that working collaboratively through an online environment can support problem-solving skills of post-primary mathematics students.
Findings demonstrated that in the Google Doc environment...

- Students were reflecting and/or trying again.
- They were using words to describe what they were doing.
- They applied a multi-representational approach to problem solve.
- The Google Doc environment supported scaffolding by the teacher and others in the group.
- They were working collaboratively.
Evidence of a willingness to accept that solving a problem involves work and time; that it is important to reflect on a problem and persist with it - displaying a productive disposition
The difference between the blues always go up in twos. The first amount of blues are 8, then 10, so on and so forth. So our formula is.. (me and ? solved this together)

\[2x + 6\]

\(x =\) the pattern we are looking for, in this case, 5. So we multiplied it by two because it goes up in twos each time. And then we plus 6 because that’s where it starts after you also take away the two blue that go with the yellow.

Check:
- Let’s do it with 5: \(2(5) + 6 = 2 + 6 = 16\)
- Let’s do it with 100: \(2(100) + 6 = 200 + 6 = 206\)
- Let’s do it with 200: \(2(200) + 6 = 406\)

Teacher encouraged students to develop a strategy for checking their answer (SEC, 2015a)
• Students were conscious that the others in their group would be looking at their work; this motivating factor causes them to reflect more through explaining the way they solved a problem.

• The comments tool allowed the teacher to remind students to check.

“On the google doc ... I always wrote out a lot of information on why I was doing stuff”

“I was aware unlike in my copy that people were gonna be looking at it... it’s just casual in your copy it’s just you were as in the google doc its more people so you have to explain it to everyone so they know what you are doing.”
Using words

When solving a problem, I describe how I solve the problem using words

<table>
<thead>
<tr>
<th></th>
<th>% of students (n=29)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agree</td>
<td>40</td>
</tr>
<tr>
<td>Neither Agree</td>
<td>30</td>
</tr>
<tr>
<td>Strongly Agree</td>
<td>20</td>
</tr>
<tr>
<td>Strongly Disagree</td>
<td>5</td>
</tr>
<tr>
<td>Disagree</td>
<td>10</td>
</tr>
</tbody>
</table>

PRE  POST

I say what is the maximum area he can have with 20m fencing.

First I found out what 4 numbers add up to twenty since a rectangle has 4 sides:
5+5+5+5 = 20
9+9+1+1 = 20
6+6+4+4=20...........ETC
It said area and to get area you have to multiply length by width.

So I figured out what length x width equaled the highest number and so I got 5x5=25 so I think the highest possible area would be 25m squared. Although you may consider it a square but we learned that a square has all the qualifications to be a rectangle: it has 4 sides and corners plus all right angles so therefore a square is a rectangle but a rectangle is not a square.

So me and ... done this the other day on her group so i think i know how to do it. I'll show you what we did.

So already we can see that each time the yellows add one and the blues add two. So if we were doing a formula we would know that no matter what whatever one we’re looking for will always be the same amount of yellows. For the first one there’s eight blues so if I’m looking for no.5 I would do 2(x) + 8 because 2 is the difference in each one. So wait let's go back to yellow. The formula for that would be x(1) (x being whatever pattern we’re looking for) nothing because the first one is just one so for blue its the same thing so 8 is like the starting amount except 8 isn't the starting amount because for each yellow there's two blue so for the first one you take away two so the formula for the blue is 2x +6 does this make any sense?
• It was because other people were viewing their work, students felt the need to explain what they were doing.

“You’re not going to write in your copy you are just write examples in your copy”

“I think that using words helped to describe what I was doing and how other people can understood the problem better”

A student came up with a formula. Another student asked her how she got the formula. This encouraged the first student to explain in words what she did.

Q3. The 100th Staircase:
\[
\frac{100(100+1)}{2} = \frac{100 \times 101}{2} = \frac{10,000}{2} = 5,050
\]

Q4. This is a quadratic pattern because the 2nd changes are constant. They are all going up in 1. It’s not a linear pattern because the 1st changes are not constant.

This is amazing! How did you figure this out??

example:

\[
x(x + 1) = \frac{2(n+1)}{2} \Rightarrow \frac{2 \times 3}{2} = \frac{6}{2} = 3
\]

At the start we add 2(pattern) + 1(change). We get 3 so then we multiply 2(pattern) = 6. We put 6 over 2 getting \( \frac{6}{2} \), so then we divide 6 by 2 and we get the answer 3.
Multi-representational

Problem: One side of a rectangle is 4m bigger than the other side. The area of the rectangle is 60 metres squared. What is the perimeter of the rectangle?

First I think we should make a table:
1 x 60
2 x 30
3 x 20
4 x 15
5 x 12
6 x 10
Then maybe from this we can figure out what the perimeter is. You’ll have to take the two numbers away until you get a difference of 4 like e.g. 30 - 2 = 28. The answer is 6 x 10 and the difference between 6 and 10 is 4. Yay.
Answer: 10 + 10 + 6 + 6 = 32 m²

This student has correctly solved the problem by creating a table and describing what she did in words.

She then drew a diagram using the handwriting tool.

Another student starts the problem using algebra, again using the handwriting tool.

\[ x(x + 4) = 60 \]
\[ x^2 + 4x = 60 \]
\[ x^2 + 4x - 60 = 0 \]
\[ (x + 10)(x - 6) = 0 \]
\[ x = -6 \text{ or } 6 \]

A student uses the g(Math) feature in Google Docs to insert an algebraic equation.

6(squared) + 4(6) = 60
36 + 24 = 60

The student who originally solved the problem now wants to go back and see if she can solve it a different way. The teacher comments and encourages her to keep going.
• It was the collaborative nature of the online environment that encouraged students to demonstrate different ways to solve a problem.

“Well one person in our group would maybe write a paragraph and explain it and then another person would go and do like in a table and then another one would do in a graph so it was kinda good to see all the different ways and if you got one way and still no one was on you'd try and find another way”
"Viewing each person's perspective like what they said and you can add on... add stuff to it... to make it better”
“I think X started a problem and I finished it”

“It was slightly confusing what to do on the 4th problem but then like X gave me a brief understanding of the problem so I managed to solve the problem on my own even having like you at home (the teacher) and it just felt more like we were learning more in-depth from the class”

• Allows for formative feedback (NCCA, n.d.). According to Hattie (1999), feedback is the most important thing a teacher can do to enhance student achievement; it’s about helping students knowing how and why they understand or misunderstand, “what directions the student must take to improve” and “matching the next teaching act to the present understandings of the student”
Does everyone agree with me on this? This is the area of a 20m rectangle but I'm not too sure what the maximum rectangular means. Does it mean the largest numbers you can multiply together to make the area? Or am I totally wrong?

Student A does not understand the problem. The teacher tells her she is correct about area (when you multiply the numbers). However, the teacher holds back from telling her everything and waits to see if another student responds.

Student B responds to student A. She recognises the misconception that student A had with finding largest numbers, not area. This is an example of peer-to-peer scaffolding. She doesn’t tell her the answer but instead points out the misconception and points her in the right direction.

Student B creates the table. The teacher challenges her to draw a graph. Students have not drawn quadratic graphs before. This is tapping into the ZPD of the student.

Student B figured out that the pattern was quadratic, something that wasn’t required in this problem. This is clear evidence of further learning and would not have happened without intervention of more knowledgeable other – “the teacher” who asked her to draw a table. Student B draws the graph and explains what she has done. Student A comments underneath that she understands.

<table>
<thead>
<tr>
<th>Length (m)</th>
<th>Width (m)</th>
<th>Total (m)</th>
<th>Area (m²)</th>
<th>Changes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
<td>20</td>
<td>9m²</td>
<td>+7m</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>20</td>
<td>16m²</td>
<td>+5m</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>20</td>
<td>21m²</td>
<td>+3m</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>20</td>
<td>24m²</td>
<td>+1m</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>20</td>
<td>25m²</td>
<td>-3m</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>20</td>
<td>24m²</td>
<td>-1m</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>20</td>
<td>21m²</td>
<td>-3m</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>20</td>
<td>16m²</td>
<td>-5m</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>20</td>
<td>9m²</td>
<td>-7m</td>
</tr>
</tbody>
</table>

I noticed that this pattern is a quadratic pattern. After I made a graph I noticed that the maximum rectangular area that Farmer Joe can fence is 20m². The changes would first increase, but after I got to the maximum rectangular area 20m², the changes would decrease.

I get what you’re doing Joe. It’s a good way to show all the different types of areas. I like it.
Collaborating

• Collaboration supports problem solving (Kim & Hannafin, 2011; Tanner & Jones, 2000)

• The Google Doc online environment allows students to communicate and help each other

“I think it was better for more independent learners that usually would study by themselves or write down everything in their copy, it was better for them to interact with other people and see that there is more than one way they do it”

• Collaboration is an important key skill; Junior Cycle, Digital Strategy, Project Maths curriculum
\[
\frac{3}{4} (x) + 3x = 65 \\
X \text{ all by } 4 \\
1(4x) + 12x = 260 \\
4x + 12x = 260 \\
16x = 260 \\
\text{if } x = 16 \frac{1}{2} \\
\frac{3}{4}(16 \frac{1}{2})
\]

A student makes a mistake and another student helps her in the comments.

Students helping each other in the Google Doc – they are typing in different colours.

Well, I was thinking that maybe 20m is the total perimeter of the rectangle so if you were to divide it by four (because a rectangle has 4 sides) you would get 5 BUT it’s a rectangle so it couldn’t be 5.

Ms said that a square is also a rectangle since a rectangle and square are shapes in which the opposite sides are equal so we can use 5x5.

A student thinks that a square is not a rectangle. Another students addresses her misconception.
Motivation

• The collaborative online environment introduced a level of competition and raised the level of expectation of what an individual can achieve..

“It helped me ... to be able to get formulas for different harder problems”

• Students pushing themselves to learn new things. Student learning how to solve a problem using Logs (Leaving Cert standard)
I only answered (ii) I am not sure if this is right, I looked at the website and tried to do it. The top part of the page is just an example. The bottom part is the question.

Student A: Trial and Error method

Student B: Using Logs method
Whole school approach to embedding problem solving

- Numeracy initiative
- Weekly puzzle competition for all students in the school
- New puzzle hung up every week
- Students submit their solutions to me on paper or by email
- Students have to describe how they get their answers
- The word “maths” not used
- No pressure put on students or teachers
- Teachers (non Maths) doing it with their students during tutor time
- Weekly prizes
- Prizes will be awarded at the end of the year
Puzzle 6

What weight will the fourth scale display?

Think carefully! There's a quick and clever shortcut for solving this.
I forgot to write on the answer I gave you that its 2D because $20 + 24 - 10 = 34$ (2 Dogs) I don't know why I said substitute and I guess it still works but I was looking at it confused and found a way to summarize it:

- $20$ (rabbit & dog) + $24$ (cat & dog) - $10$ (cat & rabbit) = $34$ (2 dogs)
- $34 / 2 = 17$
- Dog's weight = $17$
- $17 + 10 = 27$ (dog + rabbit + cat)

The - 10 Balances it out to 2 dogs (34)

Sorry about that I was going through it with someone I meet at The Walton Club and he said I did it weird so I went through it again the other way shows all the animals individual weights though so this is more of an add on then full answer again sorry about that.

Hi Ms Stone

I looked at the sheet today and saw that I got the Question wrong. I asked [name] which answer she got and she got also 30. I gave my sheet on the Friday before the exams started. I had to ways of answering it (one with my guess and one the algebraic way)

I just want to check if I got it right or wrong because I was really interested in that Question.

-I hope I caused no inconvenience but it was just out of curiosity -

Thanks,
“Today the language support programme was visited by two inspectors... they were in A4 for about an hour to chat to some of the students, see how the programme works, look at projects etc... anyway, at one stage they looked around and realised that about 8 students (some first and second years, a fifth year) in different corners of the room were all working on this week's puzzle, some with pen and paper, some using mini white boards, a couple on laptops. No one had told them to do it... it was just that when two students took the puzzle down from the whiteboard and started on it, and then others saw them working on it, they wanted to too..... They were blown away - they said how great it was to see students working so enthusiastically on maths... and especially because they had chosen to do it, not been told to.”
3 apples = 30 so we divide 30 by 3 = 10 so each apple = 10

Apple = 10 + Bananas + Bananas = 16

Since we know an apple = 10
18 - 10 = 8 ÷ 2 (pairs of bananas) = 4

Bananas = 4 which happens to be the amount in the image.

4 Bananas - 2 half coconuts = 2

Since we knew from the last question in the image Bananas = 4 so
4 - 2 = 2 = 2 half coconuts ÷ 2 = 1 half coconut

Apple = 10
Banana = Amount in image (4 in last question)
Half coconut = 1

Half coconut + Apple + Pair of 3 bananas

Bananas = 8 because when compared to other image there is 1 less banana.

so... 1 + 10 + 3 = 14
600 + ? = 1000
600 + 400 = 1000

400 + 400 = 800
So
1000 - 800 = 200g

The empty jar weighs 200g.

The answer is 200g because if a full jar of honey holds 1000g, and half a jar holds 500g, you have to take a certain amount away that will go into the bought numbers evenly. A = 200g.
Creativity

\[ x = 360^\circ - 90^\circ - 90^\circ - 30^\circ - 30^\circ = 90^\circ \]

What I did was I found the angles at the point where the two pencils met. The \( x \) angle is found using the 360° circle and the other angles.
“I entered the competition because when I looked at the first puzzle, I was immediately interested in it... What I like about the puzzle competition is that first it looks easy but actually it isn't always as easy as it looks!(fruit puzzle)”
References


